

## Titles and Abstracts

December 9, 2024, Monday

Chair: Stanislaw Janeczko, Warsaw University of Technology

### Notes on singularities theory of mappings over dual numbers

Goo Ishikawa(石川刚郎)

Hokkaido University

Two dimensional commutative and associative  $\mathbb{R}$ -algebras are classified into the complex numbers, the split complex numbers and the dual numbers, where the imaginary unit  $i$  satisfies  $i^2 = -1$ ,  $i^2 = 1$  or  $i^2 = 0$  respectively. In this presentation, we explain differential calculus and singularity theory over dual numbers, for example, dual Morse lemma, dual transversality theorem, dual division theorem and dual preparation theorem.

Chair: Huaiqing Zuo(左怀青), Tsinghua University

### Singular submanifolds in non-flat space from Legendrian dualities viewpoint

Liang Chen(陈亮)

Northeast Normal University

We investigate the geometrical and topological properties of singular submanifolds in non-flat space, such as singular curves in hyperbolic space, singular surfaces in sphere space and so on, from the viewpoint of Legendrian dualities which developed by Izumiya.

### On stringy invariant of $\mathbb{Q}$ -Gorenstein varieties via $\mathbb{Q}$ -embedded resolution

Yifan Chen(陈亦凡)

Tsinghua University

The stringy E-function was introduced by Batyrev as a new invariant of singularity to study topological mirror symmetry and to prove a version of McKay correspondence. In this talk, We give a new method of computing the stringy E-function of  $\mathbb{Q}$ -Gorenstein variety via  $\mathbb{Q}$ -embedded resolution. As applications, we calculate the stringy E-function

of a hypersurface in  $\mathbb{C}^n$  defined by a semi-quasi-homogeneous polynomial.

**Chair: Chunping Zhong( 钟春平 ), Xiamen University**

## **Effective bounds for singularities on Fano fibrations**

**Bingyi Chen( 陈炳仪 )**  
Sun Yat-sen University

Given a fibration, a natural and important question in birational geometry is to relate the singularities of the base space and those of the total space. Last year Birkar proved a conjecture of McKernan and Shokurov which says that, for all Fano fibrations from  $X$  to  $Z$  of given relative dimension, the minimal log discrepancy of singularities of  $Z$  is bounded from below by a positive number depending only on the minimal log discrepancy of  $X$ . In this talk, I will discuss the optimal bound in this result for the relative dimension one case and an effective bound for the toric case.

## **The singularities and geometrics of evolutoids of non-lightlike surfaces in Minkowski 3-space**

**Yanlin Li( 李彦霖 )**  
Hangzhou Normal University

In this talk, I will present the concept of evolutoids of non-lightlike surfaces as an envelope of a two-parameter family of lines in Minkowski 3-space. From the perspective of singularity theory, I will talk about the classification of singularities of evolutoids of non-lightlike surfaces in Minkowski 3-space. This is a joint work with Jing Li.

## **Low-Dimensional Tori in Calogero-Moser Systems**

**Guorui Ma( 马国瑞 )**  
Tsinghua University

In this talk, for the classical Calogero-Moser systems for  $SU(n)$ , we use action-angle variables to describe all degenerate Liouville tori of dimension  $1 \leq k \leq n$ . This is joint work with Andrii Lyashik, Nicolai Reshetikhin, Ivan Sechin. For the spin Calogero-Moser systems  $SU(n)$  of rank 2, we also wish to give a description of the low-dimensional Liouville tori for some certain special cases, this is a unfinished work joint with Kai Jiang and Nicolai Reshetikhin.

December 10, 2024, Tuesday

Chair: Stephen S.-T. Yau( 丘成栋 ), Tsinghua University & Beijing Institute of Mathematical Sciences and Applications (BIMSA)

## Lifting of ideals in positive characteristic to those in characteristic 0

Shihoko Ishii( 石井志保子 )  
The University of Tokyo

In the talk, I will show an appropriate lifting of an ideal plays the role of a bridge between the singularities in positive characteristic and those in characteristic zero.

Chair: Kentaro Saji( 佐治健太郎 ), Kobe University

## Reflexive modules on quotient surface singularities

Agustin Romano-Velazquez  
Institute of Mathematics, UNAM

Let  $(X, x)$  be a normal surface singularity and denote by  $L$  its link. The first complete classification of the finite dimensional representations of the fundamental group of  $L$  was done by McKay in the case of rational double point singularities. Later, Artin and Verdier, reformulate the McKay correspondence in a more geometrical setting. Their correspondence gives a complete classification of the indecomposable reflexive modules. In the case of quotient surface singularities, Esnault classified all the reflexive modules of rank one. Moreover, Esnault proved that quotient surface singularities are the only surface singularities with a finite number of indecomposable reflexive modules, such singularities are called Cohen–Macaulay finite representation type.

In this talk, we classify all the reflexive modules on quotient surface singularities. For this, we will use the Atiyah–Patodi–Singer theorem and the theory of secondary characteristic classes to construct our classification. As a consequence, the classification problem of reflexive modules over surface singularities of Cohen–Macaulay finite representation type is completely finished.

Joint work with José Antonio Arciniega-Nevárez and José Luis Cisneros-Molina.

**Chair: Jianfei Wang(王建飞), Huaqiao University**

## **Invariants of a hypersurface singularity by Waring decomposition**

**Beihui Yuan(袁北慧)**

Beijing Institute of Mathematical Sciences and Applications (BIMSA)

We consider analytic invariants of a hypersurface singularity given by the power sum decomposition of its tangent cone. Classification by the rank of Hessian matrices works for singularities with order 2. Our method can be viewed as a generalization of this method to the cases when the singularity has higher orders. Those invariants provide necessary conditions for contact equivalence.

## **Singularities of the intersection of parametric surfaces and its applications**

**Jinsan Cheng(程进三)**

Academy of Mathematics and Systems Science, Chinese Academy of Sciences

We present a complete method to compute the topology of the intersection of two parametric surfaces. We analyze the singularities of the intersection in two spaces(the parametric space-4D and the model space-3D) and point out the relationship of the points between two spaces. Furthermore, we give a method to approximate the intersection for Computer-Aided Design.

## **Approximation of spherical convex bodies of constant width**

**Huhe Han(韩呼和)**

Northwest A&F University

A theorem of Blaschke says that for every convex body of constant width  $\tau$  in the Euclidean plane  $E^2$  and every  $\varepsilon > 0$  there exists a convex body of constant width  $\tau$  whose boundary consists only of pieces of circles of radius  $\tau$  such that the Hausdorff distance between the two bodies is at most  $\varepsilon$ . Lassak presented an analog of this theorem for bodies of constant width  $\tau < \pi/2$  on the sphere  $\mathbb{S}^2$ . In this talk, we show that (1) for any spherical convex body  $C$  of constant width  $\tau > \pi/2$ , there exists a sequence  $\{C_i\}$  of convex bodies of constant width  $\tau$ , whose boundaries consist only of arcs of circles of radius  $\tau - \pi/2$  and great circle arcs such that  $\lim_{i \rightarrow \infty} C_i = C$  with respect to the Hausdorff distance; (2) for any spherical convex body  $C$  of constant width  $\pi/2$ , there exists a sequence  $\{P_i\}$  of spherical polytopes of constant width  $\pi/2$  such that  $\lim_{i \rightarrow \infty} P_i = C$  with respect to the Hausdorff distance;

December 11, 2024, Wednesday

Chair: Goo Ishikawa(石川刚郎), Hokkaido University

## Intrinsic singular points and curvatures of piecewise-smooth surfaces and their applications

Miyuki Koiso(小磯深幸)  
Kyushu University

We study piecewise-smooth (PS in short) surfaces in the three-dimensional Euclidean space, which are two-dimensional topological manifolds made by connecting finitely many smooth surfaces. We discuss intrinsic singular points of such surfaces and give new definitions which represent curvature and sharpness at each point in the 'edges' and at each 'vertex' of such a surface. These concepts are defined intrinsically by using a generalization of the classical Bertrand-Puiseux Theorem, which gives a power series expansion of the length of the geodesic circle with respect to the radius. Then, using the new concepts mentioned above, we represent the well-known Gauss-Bonnet Theorem that gives a relationship between curvatures and topology for surfaces. We discuss also the definition and characterization of PS developable surfaces which are isometric to 'planar surfaces'. And we mention a rigidity problem of such surfaces that is whether a PS closed developable surface can be isometrically-deformed preserving the enclosed volume.

Chair: Xun Yu(余讯), Tianjin University

## Schwarz lemma for Finsler metrics on the classical domains

Chunping Zhong(钟春平)  
Xiamen University

Around 1957-1958, K. H. Look studied the Schwarz lemma and analytic invariant on the classical domains of type I-IV with respect to the Bergman metrics. Recently we showed that each one of the classical domains of type I-IV (with rank greater than or equal two) admits infinitely many holomorphic invariant strongly pseudoconvex complex Finsler metrics which are all proved to be Kähler-Berwald metrics, but they are not necessary Hermitian quadratic. In this talk, we shall establish a Schwarz lemma for holomorphic invariant Kähler-Berwald metrics on the classical domains of type I-IV.

## Motivic Principal Value Integral for Hyperplane Arrangements

Quan Shi( 石泉 )  
Tsinghua University

Motivic Principal Value Integral (PVI) is defined for multivalued-rational forms on smooth complex varieties. It is the residue of the motivic zeta function and hence its vanishing and non-vanishing mean a lot. Denef, Jacobs, and Veys conjectured a geometrical vanishing implies the vanishing of PVI. In this talk, we shall give an introduction to PVI and provide our recent results about PVI for hyperplane arrangements. It is a joint work with Nero Budur and Huaqing Zuo.

**December 12, 2024, Thursday**

**Chair: Miyuki Koiso( 小磯深幸 ), Kyushu University**

## Singular points of generalized symplectic mappings

Stanislaw Janeczko  
Warsaw University of Technology

Symplectic relations, or generalized symplectic mappings, are defined as the Lagrangian submanifolds of product symplectic manifold. Canonical stratification of the category of linear symplectic relations is constructed and applied to searching the special vertical points (singular points) of general symplectic relations. The theorem on invariant decomposition of linear symplectic relations onto canonical reductions and symplectomorphisms is presented. Extension of the action of symplectic mappings is used to investigation of billard map closed trajectories and cohomological invariant of the discrete dynamical systems.

**Chair: Agustin Romano-Velazquez, Institute of Mathematics, UNAM**

## On automorphism groups of smooth hypersurfaces

Xun Yu( 余讯 )  
Tianjin University

We show that smooth hypersurfaces in complex projective spaces with automorphism groups of maximum size are isomorphic to Fermat hypersurfaces, with a few (explicitly given) exceptions. This is a joint work with Song Yang and Zigang Zhu.

## On the Nakai Conjecture for some singularities

Zida Xiao(肖子达)

Tsinghua University

The well-known Nakai Conjecture concerns a very natural question: For an algebraic variety, how does the differential operators of its coordinate ring imply the smoothness of it? It has been shown that all higher derivations of a smooth complex variety can be generated by the first order derivations, and Nakai proposed the converse question: if the algebra of differential operators is generated by the first order derivations, is the variety smooth? In this talk, I will introduce the history and present our work on the cases of isolated homogeneous hypersurface singularities and small modality hypersurface singularities.

**Chair: Xiankui Meng(孟宪奎), Beijing University of Posts and Telecommunications**

## On the k-th Tjurina number of weighted homogeneous singularities

Chuangqiang Hu(胡创强)

Beijing Institute of Mathematical Sciences and Applications (BIMSA)

Let  $(X, 0)$  denote an isolated singularity defined by a weighted homogeneous polynomial  $f$ . Let  $\mathcal{O}$  be the local algebra of all holomorphic function germs at the origin with the maximal ideal  $m$ . We study the  $k$ -th Tjurina algebra, defined by  $A_k(f) := \mathcal{O} / \langle f, mJ(f) \rangle$ , where  $J(f)$  denotes the Jacobi ideal of  $\mathcal{O}$ . The zeroth Tjurina algebra is well known to represent the tangent space of the base space of the semi-universal deformation of  $(X, 0)$ . Motivated by this observation, we explore the deformation of  $(X, 0)$  with respect to a fixed  $k$ -residue point. We show that the tangent space of the corresponding deformation functor is a subspace of the  $k$ -th Tjurina algebra. Explicitly calculating the  $k$ -th Tjurina numbers, which correspond to the dimensions of the Tjurina algebra, plays a crucial role in understanding these deformations. According to the results of Milnor and Orlik, the zeroth Tjurina number can be expressed explicitly in terms of the weights of the variables in  $f$ . However, we observe that for values of  $k$  exceeding the multiplicity of  $X$ , the  $k$ -th Tjurina number becomes more intricate and is not solely determined by the weights of variables. In this paper, we introduce a novel complex derived from the classical Koszul complex and obtain a computable formula for the  $k$ -th Tjurina numbers for all  $k \geq 0$ . As applications, we calculate the  $k$ -th Tjurina numbers for all weighted homogeneous singularities in three variables. This is a joint work with Stephen S.-T. Yau and Huaqing Zuo.

## On the Yau sequence over star-shaped dual resolution graph

Fanning Meng( 孟凡宁 )  
Guangzhou University

In this paper, we study the Yau sequence concerning the minimal cycle over star-shaped dual resolution graph, and consider the relations between the minimal cycle  $A$  and the fundamental cycle  $Z$ . Further, we also give the coincidence between the canonical cycles and the fundamental cycles from the Yau sequence concerning the minimal cycle.

## Geometry of Gluing Developable Surfaces

Junzhen Li( 李俊臻 )  
Kobe University

In this talk, it will be presented that the developable surface obtained from the frame defined by the unit vectors along the gluing curve where two surfaces are glued and provides the conditions for this developable surface to be in the special case.

**December 13, 2024, Friday**

**Chair: Shihoko Ishii( 石井志保子 ), The University of Tokyo**

## Normal form of the central singularities of D4-bifurcation of fronts and its applications

Kentaro Saji( 佐治健太郎 )  
Kobe University

A form representing a singular point is an  $SO(3)$ -normal form if it covers the A-equivalent class of the singular point by a coordinate change on the source space and an isometry on the target space.

In this talk,  $SO(3)$ -normal forms of the D4-singularities of fronts in the three space will be presented.

As an application, local and global differential geometric properties of the D4-singularities will be presented.



**Chair: Jinsan Cheng(程进三), Academy of Mathematics and Systems Science,  
Chinese Academy of Sciences**

## **Flops connecting minimal models**

**Yifei Chen(陈亦飞)**

Institute of Mathematics, Chinese Academy of Sciences

Minimal models are one of outcomes of Minimal Model Program (MMP). When running MMP, minimal models are not unique. It is natural to ask that what are the relations among these minimal models? It is known that flops connect these minimal models. We shall show that any two minimal models of lc algebraically integrable foliated triples on  $\mathbb{Q}$ -factorial klt varieties are connected by a sequence of flops. This is a joint work with Jihao Liu and Yanze Wang.

## **Geometry on curves passing through Whitney umbrella**

**Hiroyuki Hayashi(林弘幸)**

Kobe university

Whitney umbrella is a singularity which appears most frequently on surfaces. To study differential geometry on a surface, a unit normal vector plays a central role. However, a unit normal vector on Whitney umbrella is not extended. In this talk, we show that we can extend a unit normal vector beyond a Whitney umbrella if we consider a curve passing through Whitney umbrella. Then a study of geometry by using invariants defined by a Darboux frame consisting of smoothly defined unit normal and tangent vectors along a curve passing through Whitney umbrella will be presented. Geometric meanings of the invariants will also be presented.

**Chair: Yifei Chen(陈亦飞), Institute of Mathematics, Chinese Academy of  
Sciences**

## **Bernstein-Sato roots for weighted homogeneous singularities in positive characteristic**

**Siyong Tao(陶斯咏)**

Tsinghua University

We prove that in regular  $F$ -finite rings of positive characteristic, the Bernstein-Sato root set of the tensor product of ideals is the union of their respective Bernstein-Sato root sets. Moreover, by computing some special  $F$ -thresholds, we provide an explicit description about the Bernstein-Sato roots of a weighted homogeneous polynomial with an isolated singularity at the origin in positive characteristic.

## **Circles of a Surface and their Centres**

**Graham Reeve**

Liverpool Hope University

It has long been known that we can use the contact between a surface and so called 'model submanifolds' to tell us about the underlying geometry of a curve or a surface. For example, we can tell something about how flat a surface is by its contact with either planes or lines, and something about how round it is by its contact with spheres or circles. In this talk I'll revisit the idea of contact between a surface and circles and discuss the idea of a "surface of centres" of circles with sufficiently high contact, a kind of generalisation to the focal set.

## **The weights of three types of isolated curve singularities are determined by Hodge ideals**

**Yang Wang (汪洋)**

Tsinghua University

We calculate Hodge ideals and Hodge moduli algebras for three types of isolated quasi-homogeneous curve singularities. We show that Hodge ideals and Hodge moduli algebras of the singularities can determine the weights of the polynomials defining these singularities.

We give some examples to explain why Hodge moduli algebras and Hodge moduli sequence are better invariants than characteristic polynomial (a topological invariant of the singularity) for non-degenerate quasi-homogeneous singularities, in the sense that characteristic polynomial cannot determine the weight type of the singularity. Furthermore, from the observation of some examples, we raise a conjecture that the Hodge moduli numbers of isolated quasi-homogeneous hypersurface singularities remain constant under semi-quasi homogeneous deformation.